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ECONOMIC OPTIMIZATION OF A MULTIPLE EFFECT HUMIDITY CYCLE

A THESIS

Presented to

The Faculty of the Graduate Division

by

Richard A. Collins

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

Georgia Institute of Technology

June, 1962

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ECONOMIC OPTIMIZATION OF A

MULTIPLE EFFECT HUMIDITY PROCESS

Approved:

Date Approved by Chairman May 28, 1962

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Professor Wm. A. Hinton for his encouragement and guidance and to Dr. C. W. Gorton and Dr. W. N. Grune for reviewing this thesis.

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NOMENCLATURE

A - cross sectional area of packed column, sq ft

A_{WW} - heat transfer area in the water-to-water heat exchanger, sq ft

a - ration of area of heat or mass transfer to envelope volume, sq ft/cu ft

C - packing factor of packing material, dimensionless

C_c - unit cost of packed column, dollars/cu ft

C_{hp} - unit cost of pumps or blowers, dollars/horsepower

C_{St} - unit cost of steam, dollars/1000 lb

$\overline{C}_s$ - average humid heat of air-water vapor mixture, BTU/lb ^oF

f_p - unit pressure drop for gas in packed column, psi/ft

G_x - mass velocity of water, lb/hr sq ft

G_y - mass velocity of gas, lb ga/hr sq ft; G_y' - lb dry air/hr sq ft

H - absolute humidity, lb water/lb dry air

HTU - height of a transfer unit, ft

hp - horsepower

h_x - water side heat transfer coefficient, BTU/hr sq ft ^oF

h_y - gas side heat transfer coefficient, BTU/hr sq ft ^oF

I - a particular value of enthalpy potential in the integral that describes NTU, BTU/lb

i - enthalpy, BTU/lb; i_i - of gas at interfacial conditions; i_y - of gas at bulk stream conditions; i_S - enthalpy available in one pound of steam

K - total cost of water conversion, dollars/day

k'_y - mass transfer coefficient, lb/hr sq ft

M - gas flow rate, lb/hr

M_{St} - steam rate, lb steam/day

NTU - number of transfer units, dimensionless

P - plant production, gallons/day

ΔP - pressure drop, across packing, in of H_2O

p - absolute pressure, psia

R - slope of a line describing the response of I to t_x, BTU/lb ^oF

T_{a_m} - mean bulk temperature of air, ^oF

TPI - total plant investment, dollars

ΔT_{EC} - water temperature difference between a point in the evaporator and a point in the condenser of equal air enthalpy, ^oF

ΔT_H - temperature rise in water heater, ^oF

ΔT_{WW} - temperature difference between the hot and cold liquids in the water-to-water heat exchanger, ^oF

t_x - water temperature in packed column, ^oF

t_i - temperature at the interface, ^oF

V_e - volume of packed column, cu ft

v - specific volume, cu ft/lb dry air or cu ft/lb gas

W - water flow rate, lb/hr

Z - packed column height, ft

Z' - pressure head, ft of H_2O

ρ - density, lb/cu ft; ρ_x - water density; ρ_y - gas density

μ - viscosity, centipoise

η	-	turbine efficiency, per cent
ψ	-	symbol designating an equation of restraint
λ	-	a Lagrangian unassigned multiplier

CHAPTER I

INTRODUCTION

The present cost of converting salt, or brackish water, to potable water for human consumption is too high to compete with other sources of water supply in most areas. At the present time there are many water conversion processes under development. It is generally recognized that no single process, nor any fixed design of a particular process, is most economic in all locations and applications. Therefore, there is a practical need for a general method of determining the most suitable process and design for a given set of conditions.

This study is concerned with determining the design and operating conditions of a particular process, a humidity cycle, which will allow water conversion at the lowest possible cost.

The multiple effect humidity cycle was conceived in January, 1961 as an improvement over the single effect humidity cycles previously studied at the Georgia Institute of Technology (1)(2)(3). The several designs of the multiple effect cycle studied show that a definite optimum arrangement exists, and that the cost of the product water may be economically attractive for such an arrangement.

The cycle under study is basically comprised of three heat and/or mass transfer processes. In each process the transfer potential may be related to a temperature difference. From the standpoint of thermodynamics temperature differences of small magnitude are desirable in that the efficiency of the cycle may then approach that of a reversible cycle, while from the standpoint of economics, temperature differences of large magnitude are desirable because for a given transfer coefficient the equipment size is inversely proportional to the transfer potential. The primary objective of this study is to balance the cycle efficiency with the equipment economics to give an optimum unit cost for product water.

This optimization may be accomplished by formulating expressions relating to the performance variables, and using them as constraints in optimizing a cost function for the overall process. Differential calculus was selected as the technique for carrying out the optimization calculations on the grounds that the resulting procedure may be applied to other unit processes owing to the general nature of differential calculus.

It is interesting to note that the literature is vacuous of attempts at economic optimization of processes similar to the humidity cycle by the methods of differential calculus. The reason for this, in many cases, is the lack of a mathematical model to represent the process. In such cases the designer is forced to resort to the more tedious statistical methods.

C. S. Hsing (4) treated two fractionation processes by statistical methods including: the Response Surface Method; and the Slope Comparing Method. Each process was optimized with respect to two operating variables. The Flour Corporation (5) made a study of a multistage flash evaporation process for sea water conversion in which several variables of operation as well as several variables of construction were optimized by independent variation of the variables. Trial and error techniques were used to finally coordinate the optimum values of the several variables.

Other investigators (6)(7) report studies of performance characteristics and economic analysis for water conversion processes. However, no attempt at economic optimization was made.

An excellent example of the application of differential calculus, by means of Lagrange's method of undetermined multipliers for minimizing costs is presented by Happel (8) in which he determines the optimum design of a high-pressure gas-transmission system.

CHAPTER II

DESCRIPTION OF THE MULTIPLE EFFECT HUMIDITY CYCLE

Figure 1 shows the basic cycle, consisting of a heat source, brine evaporators and condensers. The brine is passed through a heater where it experiences a temperature rise, then through packed towers where water vapor and heat are given up to a counter-current air stream, reducing the brine temperature. The reduction in brine temperature in the evaporator and the temperature increase across the heater determine approximately the number effects,* as well as equipment size. One packed tower, or several in series, may be used depending on design conditions, however, air must be bled off into the condenser at various points for efficient operation. After leaving the packed towers a portion of the brine is discharged as blowdown to prevent salt build-up, followed by the addition of sea water equal in amount to the blowdown plus the fresh water production. The brine feed temperature and quantity, as well as the amount of blowdown must be such as to lower the temperature of the brine stream to the proper level, otherwise a violation of the second law of thermodynamics occurs in the condenser. The brine then enters the condenser where it cools the air stream from the evaporators. The distillate from the condensers is collected as fresh water, while the brine returns to the heater.

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*The exact number of effects may be determined by the ratio of the plant production times the latent heat of vaporization divided by the total heat input.

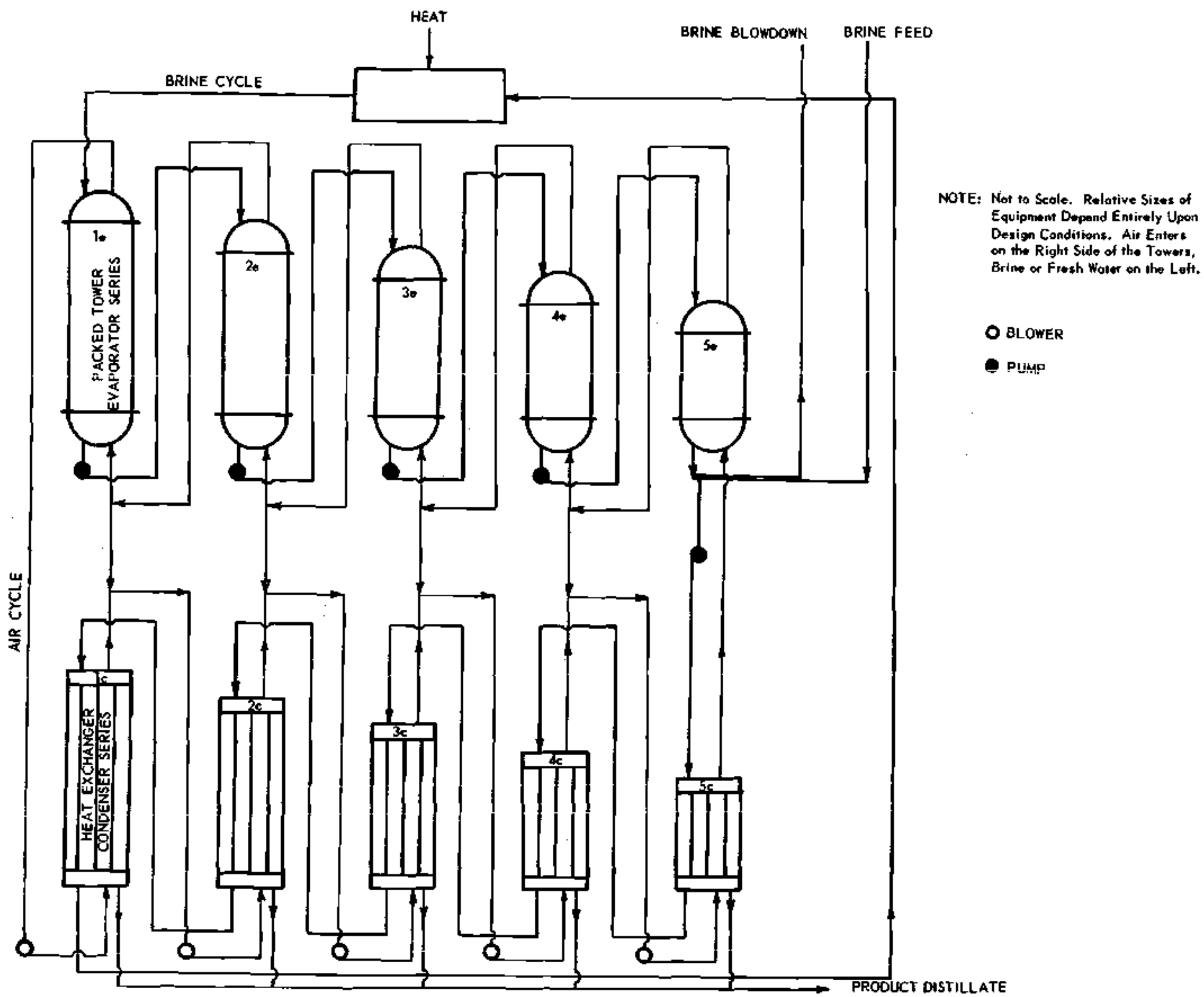

Figure 1. Multiple Effect Humidity Cycle.

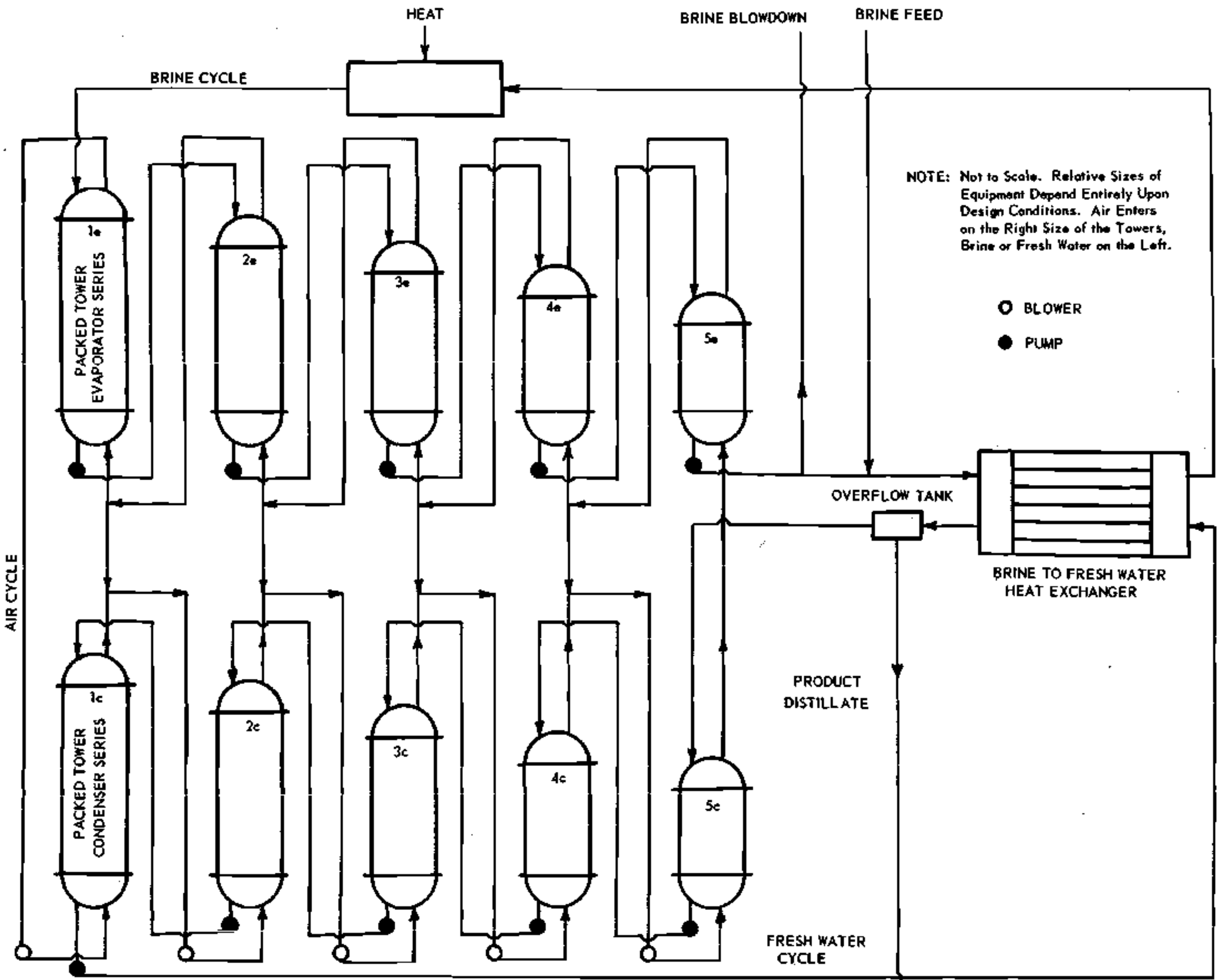

Figure 2. Variation of Multiple Effect Humidity Cycle.

Figure 2 shows one of the many possible variations of the cycle, in which packed towers and a brine to fresh water heat exchanger are substituted for the condensers. Since the condenser heat transfer surface requirement is usually quite large in a humidity system, this variation may have some merit, despite increased operating costs.

While specific descriptions of equipment to be used in the process are not available at the present, due to the dependence of size and construction on operating conditions, the following general descriptions are included for clarification:

1. Packed Towers -- Packed towers or columns consist of upright shells filled with loose pieces of solid material of uniform size thrown in at random. The packing material is generally of such size and shape as to provide a high contact surface and a low pressure drop. Examples of such packing are: Raschig rings, Berl saddles, Lessing rings, and Prym rings. Of special interest in this study is the use of "MASPAC" FN-200 plastic tower packing.* Liquid distribution plates and packing support plates are necessary in large packed towers.

2. The Heat Absorber -- This item may assume many forms, depending on the energy source, viz., a solar water heater, a steam ejector, etc.

3. Pumps and Blowers -- Fresh water pumps may be standard centrifugal pumps while brine pumps will require special materials to guard against corrosion. Blowers may be standard centrifugal blowers.

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*A product of the Dow Chemical Co., Midland, Michigan.

4. Heat Exchangers -- The water to water heat exchanger used as a basis for cost analysis in this study is the jacketed pipe exchanger consisting of a horizontal tube bundle through which the brine coolant passes in counter-current flow to the fresh water stream surrounding the tube bundle.

5. Prime Movers -- In this variation of the cycle the pump and blower drive is by direct linked steam turbine operating on purchased steam. The exhaust steam is further used in the heater as the primary heat source.

CHAPTER III

CYCLE DESIGN AND OPTIMIZATION

Introduction.-- The equations constraining the overall cost function to be optimized are obtained from the theory of heat and mass transfer mechanisms. These constraining equations consist of the variables to be optimized (Z, ΔT_{WW}, and ΔT_{EC}) and a number of constants which characterize the humidity cycle. In the following sections the constants are evaluated by making a base design. Subsequently, Lagrange's method of undetermined multipliers is applied to optimize the design.

Design Procedure.-- The design of this cycle draws heavily on the enthalpy-potential method. A detailed treatment of this theory of mass transfer is presented in the work of McCabe and Smith (9). The reader is referred to this reference for the limitations of the theory.

A cycle design is investigated by first arbitrarily choosing the number of effects desired. The number of effects is approximately equal to the temperature drop of the brine in the evaporators divided by the temperature rise given to the brine in the water heater.

Operation of the evaporating and condensing columns is shown on the Temperature-Enthalpy plot in Figure 3. Suitable temperatures are chosen at various points in the columns and air bleed-offs positioned to prevent super-saturation of the air stream (see Figure 3). The packed columns are then designed by assuming a unit pressure drop for the air

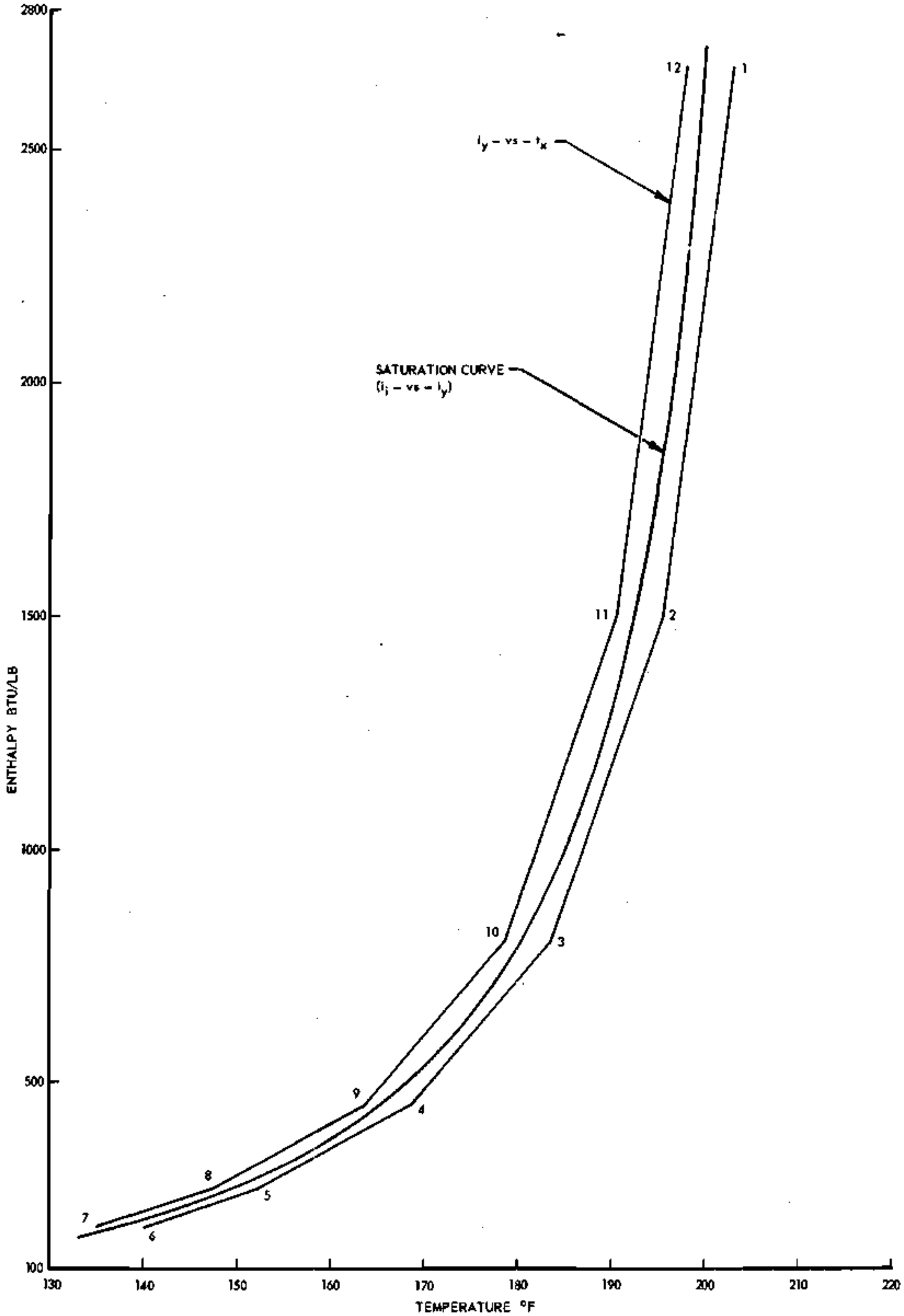

Figure 3. Operating Lines and Interface Conditions for Complete Evaporating and Condensing Cycle.

stream, determining the flooding* velocities, and continuing the calculations for equipment size and total pressure drop.

For this base design a cycle was chosen with a maximum temperature of 203°F. The temperature rise given to the brine in the heater is 10°F. The five section column for evaporation operates between the water temperatures of 203° and 140°F, thus giving approximately six effects. The condenser consists of a five section column with a water temperature operating range of 135° to 198°F. Enthalpy of the condensate is returned to the brine stream in a jacketed pipe heat exchanger with a five degree temperature difference between the hot and cold sides.

In Figure 3, lines joining points 1 through 6 are the operating lines for the evaporating process, while lines joining points 7 through 12 are the operating lines for the condensing series. The curved line between the two sets of operating lines is the saturation curve for water vapor and air at atmospheric pressure. This curve represents the interfacial conditions between the gas phase and liquid phase of the water. The assumption that the saturation curve is not materially affected by the pressures within the columns is seen to be reasonable in the course of the calculations owing to the small deviation of the pressure from one atmosphere.

The ratio of dry-air mass velocity (G_y') to liquid mass velocity in a column section is equal to the slope of the operating line and is

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*The flooding velocity is the velocity at which the pressure drop increases sharply with a slight increase in gas velocity.

fixed when the operating lines are chosen

$$G_x/G_y' = \frac{\Delta i}{\Delta t}. \tag{1}$$

Since the saturated air mass velocity (G_y) is greater than the dry-air mass velocity by a factor of one plus the absolute humidity (H), the ratio of G_x to G_y is

$$G_x/G_y = \left(\frac{1}{1 + H}\right) \frac{\Delta i}{\Delta t} \tag{2}$$

The flooding velocities may be determined from a generalized pressure drop correlation in which the group ($G_x/G_y\sqrt{\rho_x/\rho_y}$) is plotted against the group $G_y^2 \left(\frac{C\,\mu^{0.2}}{g_c\rho_x\rho_y}\right)$. Table 1 shows the values of these groups, the properties and the corresponding gas mass velocities for each section of column at flooding and at a pressure drop (f_p) of one inch per foot of column.

Velocities were selected close to those giving one inch of pressure drop per lineal foot of column length and are shown in Table 2.

With the foregoing information the size of the columns may be calculated using a graphical method due to Merkel if the individual heat transfer coefficients (h_x a and h_y a) are known. The standard equations for one inch Raschig rings

$$h_x\ a = 0.8\ (G_y')^{0.7}\ (G_x)^{0.5},\ \text{and}$$

$$h_y\ a = 2.2\ (G_y')^{0.7}\ (G_x)^{0.07}$$

were modified in proportion to the ratio of the transfer area per unit volume for "MASPAC" FN-200 to that of Raschig rings to give

Table 1 Flooding Velocities and Pressure Factor

T_{a_m}	$\frac{\rho_x}{\rho_y}$	$\frac{G_x}{G_y}$	$\frac{G_x}{G_y}(\frac{\rho_x}{\rho_y})^{1/2}$	$G_y^{2}(\frac{C\ u^{0.2}}{g_c \rho_x \rho_y})$	G_y^{2}	G_y'	Section
				Flooding			
196.2	0.0277	56.25	1.560	0.0130	0.0490	285	1
186.3	0.0288	29.65	0.852	0.0242	0.0980	574	2
172.8	0.0297	15.41	0.459	0.0350	0.1840	1040	3
157.4	0.0307	8.49	0.261	0.0680	0.3065	1550	4
143.6	0.0314	5.68	0.179	0.0870	0.4060	1950	5
				ΔP = 1 inch H_2O			
196.2				0.0100	0.0378	251	1
186.3				0.0180	0.0729	495	2
172.8				0.0310	0.1279	848	3
157.4				0.0445	0.2015	1258	4
143.6				0.0540	0.2525	1540	5

Table 2 Design Parameters

Section	G'_y	Per Cent Flooding	G_x	$\frac{h_x}{h_y}\bar{C}_s$	HTU
1	250	87.7	39,225	38.0	1.70
2	500	87.1	29,165	21.9	1.38
3	850	81.7	19,831	12.7	1.16
4	1250	80.6	13,624	8.6	1.04
5	1550	79.5	10,338	6.6	0.965

$$h_x\, a = 0.594\, (G_y')^{0.7}(G_x)^{0.5}, \text{ and} \tag{3}$$

$$h_y\, a = 1.56\, (G_y')^{0.7}\, (G_x)^{0.07}. \tag{4}$$

These coefficients may be used to determine the ratio of the enthalpy to temperature between the fluid streams and the interface for air-water contact according to the relation

$$\frac{i_i - i_y}{t_i - t_x} = - \frac{h_x}{k_y'} = - \frac{h_x\, \overline{C}_s}{h_y} = - \frac{0.594}{1.56}\, \overline{C}_s\, (G_x)^{0.43}. \tag{5}$$

If a line with the slope given by equation (5) is plotted on the enthalpy temperature plane beginning at any point on the operating line, the vertical distance between the points of intersection of equation (5) with the operating line and the saturation line gives the value of $(i_i - i_y)$ corresponding to a value of i_y equal to the enthalpy of the point of intersection at the operating line.

From an enthalpy balance around a differential element in the vertical direction (Z) it may be shown (8) that

$$\frac{h_y\, a}{\overline{C}_s\, G_y'}\, dZ = \frac{di_y}{(i_i - i_y)}. \tag{6}$$

By integration the column height (Z) between points 1 and 2 is

$$Z = \left(\frac{\overline{C}_s\, G_y'}{h_y\, a}\right) \int_1^2 \frac{di_y}{(i_i - i_y)} \tag{7}$$

The quantity multiplying the integral on the right side of equation (7) is called the height of a transfer unit (HTU) and the value of the integral is called the number of transfer units (NTU).

By the mean value theorem for integrals, the value of the integral of equation (7) may be written as

$$\int_1^2 \frac{di_y}{(i_i - i_y)} = \frac{1}{I} \; (i_{y_2} - i_{y_1}) \tag{8}$$

where I is some particular value of $(i_i - i_y)$ in the region between points 1 and 2. Evaluation of I by graphical integration of equation (8) for this base design is shown in the Appendix in Figures 8, 9, 10, 11 and 12, and Table 3.

Substitution of equation (8) into equation (7) gives a convenient expression for column height as

$$Z = (HTU) \; (\Delta i_y)/I \tag{9}$$

If the plant size is chosen to be 100,000 gallons per day, the water flow rate (W) and the air flow rate (M) may be calculated by the usual thermodynamic relations. Power requirement for the blowers and pumps, respectively, may then be determined by the standard equations

$$(hp)_B = \frac{M\,(144)(3.463)}{(33{,}000)(60)} \quad p_1\,v_1 \quad [(\frac{p_2}{p_1})^{0.29} - 1] \tag{10}$$

and

$$(hp)_p = \frac{(W)\;Z'}{(60)(33{,}000)} \tag{11}$$

where p_1 and p_2 are the intake and exhaust pressures of the blower, v_1 is the specific volume of air entering the blower, and Z' is the total pressure head of water in the water line.

Optimization Procedure.-- One of the equations of restraint needed to control the overall cost function may be developed from equation (9) of the base design. Equation (9) may be written as

$$Z = (HTU) \ (NTU) \tag{12}$$

where

$$NTU = \frac{(\Delta i_y)}{I} \tag{13}$$

The terms in equation (13) are evaluated in the base design. A general equation may be written for the number of transfer units by letting I be a function of water temperature. The type of function relating I to t may be determined by shifting the working lines of Figure 3 parallel to themselves and observing the effect on I. Figure 4 demonstrates that I is nearly a linear function of t_x and may be written as

$$I = I' + R \ (t_x - t_x') \tag{14}$$

where I' is the value of I for the base design and t_x' is the value of the water temperature corresponding to I' and the value of R may be measured as the slope of line representing i_i.

In Figure 4 the response of I to t_x was investigated at both ends and at the midpoint of section 2 of the condenser and evaporator. The values of R measured at these three points show that R measured at the

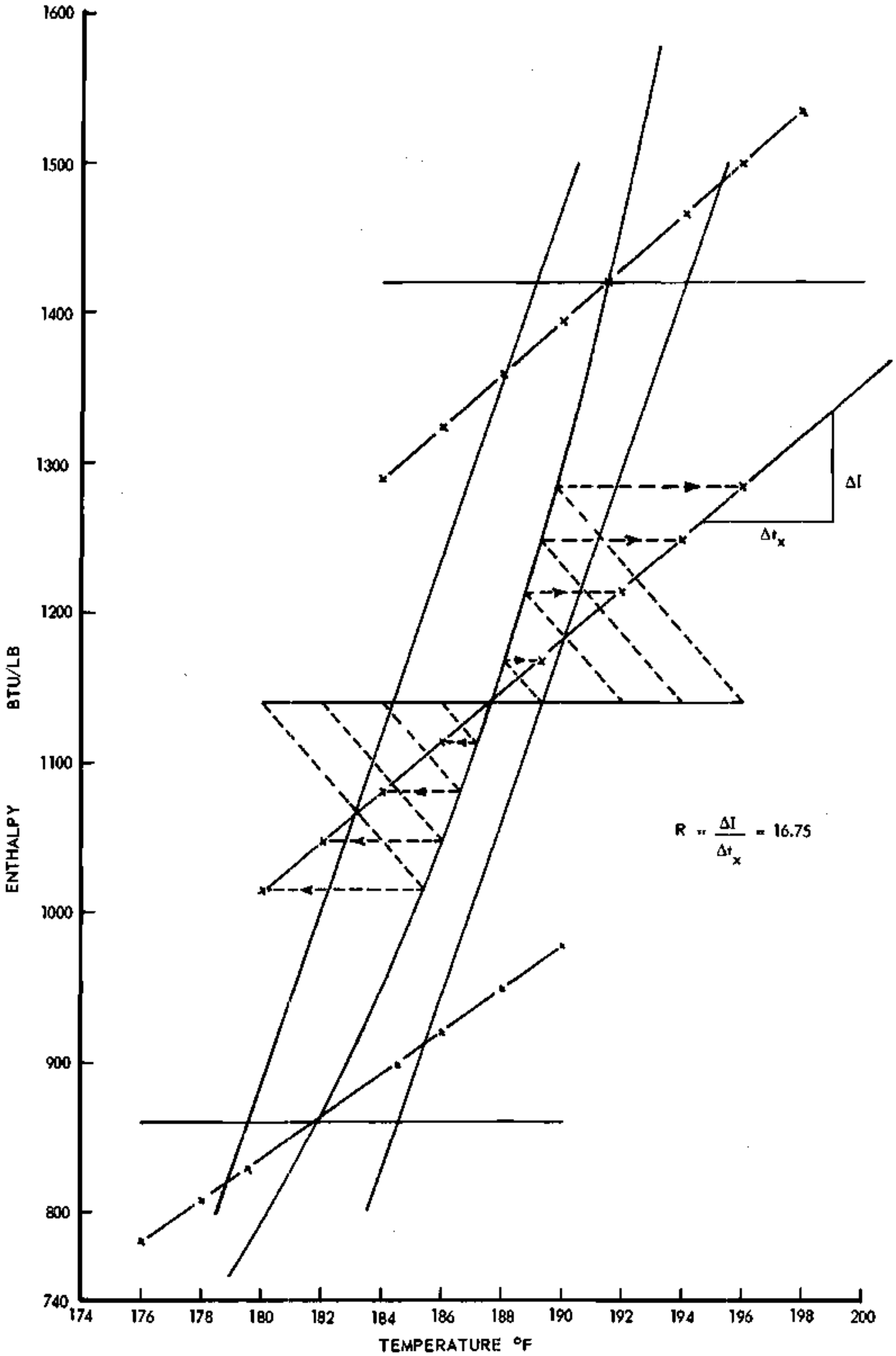

Figure 4. Response of Enthalpy Potential to Water Temperature.

midpoint is approximately the average of the R's at the endpoints. Furthermore, only a small variation of the values is found between any two of the points. Therefore R of the midpoint may be assumed to be typical of the whole section.

Equation (13) may now be generalized to represent the number of transfer units in the form

$$(NTU)_e = \frac{(\Delta i_y)_e}{I'_e + R\,(t_x - t'_x)_e} \tag{15}$$

and

$$(NTU)_c = \frac{(\Delta i_y)_c}{I'_c + R\,(t'_x - t_x)_c} \tag{16}$$

where the subscript e refers to the evaporator section and c to the condenser section.

If it is stipulated that when the working line for the evaporator is moved to the right (Figure 3) the working line for the condenser is moved an equal amount to the left, the temperature difference in equations (15) and (16) become equal and may be written as

$$t_{x_e} - t'_{x_e} = t'_{x_c} - t_{x_c} = \left(\frac{t_{x_e} - t_{x_c}}{2}\right) - \left(\frac{t'_{x_e} - t'_{x_c}}{2}\right). \tag{17}$$

Define ΔT_{EC} equal to $(t_{x_e} - t_{x_c})$.

Substitution of equation (17) into equations (12) and (13) and addition gives:

$$(NTU)_e + (NTU)_c = \frac{(\Delta i_y)_e}{I'_e + \frac{R}{2}(\Delta T_{EC} - \Delta T'_{EC})} + \frac{(\Delta i_y)_c}{I' + \frac{R}{2}(\Delta T_{EC} - \Delta T'_{EC})}. \qquad (18)$$

Since $(\Delta i_y)_e$ is equal to $(\Delta i_y)_c$, equation (14) may be reduced to a simpler form if I'_e is made equal to I'_c by proper selection of the end point temperatures in Figure 3 for the various sections. Then the total number of transfer units for both the condenser and evaporator section is:

$$(NTU)_{total} = \frac{2(\Delta i_y)}{I' + \frac{R}{2}(\Delta T_{EC} - \Delta T'_{EC})} \qquad (19)$$

The total length of column for both the evaporator series and the condenser series may be expressed as

$$Z = \sum_{j=1}^{5} \frac{2(HTU)_j (\Delta i_y)_j}{I'_j + \frac{R}{2}(\Delta T_{EC} - \Delta T'_{EC})}. \qquad (20)$$

Equation (20) is the first restraining equation formulated for the cost function and is based on considerations of heat and mass transfer. Values for the constants in equation (20) were determined in the base design and are listed in Table 4.

Table 4 Value of Constants for Equation (20)

j	$(\Delta i_y)_j$	I'_j	R_j	$(HTU)_j$
1	1177	74.50	27.45	1.70
2	700	38.07	16.75	1.38
3	350	19.53	8.375	1.16
4	180	11.57	4.786	1.04
5	80	8.117	3.313	0.965

$\Delta T'_{EC} = 5$

A second restraining equation may be taken from the temperature relations of the cycle. A simplified flow chart of the humidity cycle is shown in Figure 5 with temperatures labeled at the points of interest.

A quantity of brine beginning at the temperature T_a and flowing around the brine loop must end up at the same point with the same temperature. Thus,

$$(T_b - T_a) + (T_c - T_b) + (T_d - T_c) + (T_e - T_d) + (T_a - T_e) = 0 \quad (21)$$

For the fresh water loop

$$(T_i - T_h) + (T_f - T_i) + (T_g - T_f) + (T_h - T_g) = 0 \quad (22)$$

From Figure 3 it is seen that the water temperature change across the condensing columns is equal to the temperature change across the evaporating columns. Thus,

$$(T_c - T_b) = (T_h - T_i) \quad (23)$$

Equations (21) and (22) substituted into equation (23) gives:

$$(T_b - T_a) = - (T_g - T_f) - (T_d - T_c) - (T_e - T_d) - (T_a - T_e)$$

$$(T_b - T_a) = (T_f - T_e) + (T_c - T_g) + (T_d - T_d) + (T_e - T_a) \quad (24)$$

Since $T_e = T_a$ equation (24) reduces to

$$(T_b - T_a) = (T_f - T_e) + (T_c - T_g) \quad (25)$$

The quantity $(T_b - T_a)$ is the parameter (ΔT_H); the quantity $(T_e - T_g)$

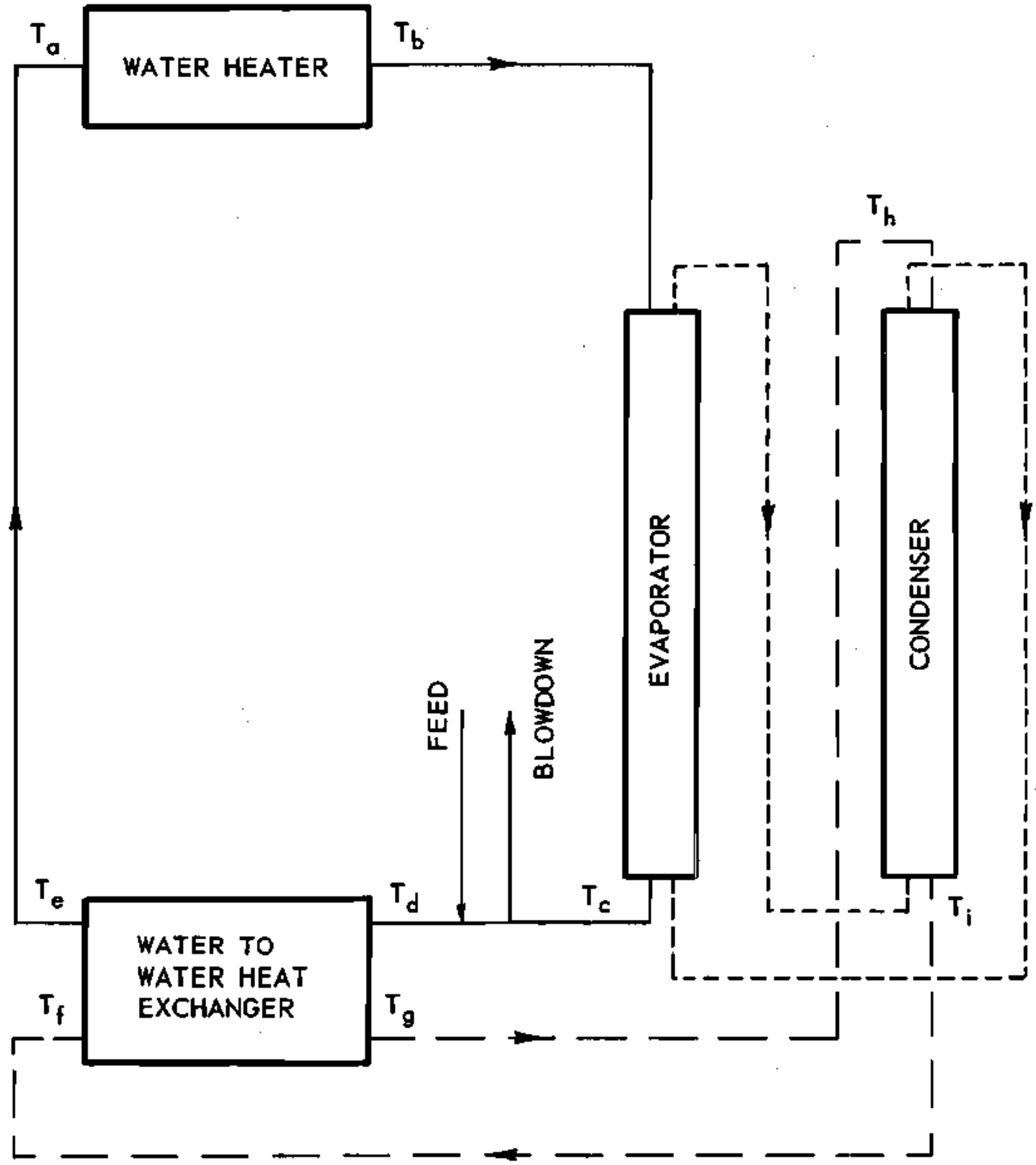

Figure 5. Simplified Flow Chart for Humidity Cycle.

is the (ΔT_{EC}) of equation (20); and the quantity $(T_f - T_e)$ is the temperature potential for the water-to-water heat exchanger (ΔT_{WW}).

Equation (25) may be written as

$$(\Delta T_H) = (\Delta T_{WW}) + (\Delta T_{EC}) \tag{26}$$

and is the second equation of restraint for the cost function.

The cost equation is made up of capital charges and operating costs including labor, maintenance, supervision, etc. This equation follows the standard procedure set forth by the Department of Interior (10) for estimating the cost of water conversion plants. The cost equation may be expressed as

$$\begin{aligned} K = {} & 1.158 \times 10^{-3} \; M_{st_T} C_{st} + 6.065 \times 10^{-4} \, [A_{WW} C_{A_{WW}} \\ & + V_c C_c + (hp)_p \, (C_{hp})_p + (hp)_B \, (C_{hp})_B] + 0.895 \, P \times 10^{-5} \end{aligned} \tag{27}$$

where K is the cost per day, M_{st_T} is the quantity of steam required, C_{st} is the unit cost of steam, A_{WW} is the heat exchange area required in the water-to-water exchanger, $C_{A_{WW}}$ is the unit cost of heat exchange area, V_c is the volume of packed column, C_c is the unit cost of packing, $(hp)_p$ is pump horsepower, $(C_{hp})_p$ is a unit cost for pump horsepower, $(hp)_B$ is the blower horsepower, $(C_{hp})_B$ is the unit cost for blower horsepower, and P is the total production of the plant for one day. Development of equation (27) in a more general form is shown in the Appendix along with the development of the unit cost equations.

Substitution of equations (A-8), (A-9), (A-11), (A-12) and (A-13) into equation (27) gives:

$$K = 2.897 \times 10^{-7} [0.0617\, WZ + 2745 \left[\left(\frac{14.7 + \frac{Zf_p}{2}}{14.7 - \frac{Zf_p}{2}} \right)^{0.29} - 1 \right] M$$

$$+ 19.2\, W\, (\Delta T_H)] + 6.065 \times 10^{-4} [39.3\, (W/\Delta T_{WW})^{0.538} + 6.6W$$

$$\sum_{j=1}^{5} \frac{C_j\, (HTU)_j\, (\Delta i_y)_j}{I'_j + \frac{R_j}{2}(\Delta T_{EC}-5)} + 0.1077\, (WZ)^{0.591}$$

$$+ 53.59\, (M)^{0.725} \left[\left(\frac{14.7 + \frac{Zf_p}{2}}{14.7 - \frac{Zf_p}{2}} \right)^{0.29} - 1 \right]^{0.725}]$$

$$+ 0.895\, P \times 10^{-5} \tag{28}$$

Equation (28) is of the form $K = K\,(Z, \Delta T_H, \Delta T_{WW}, \Delta T_{EC})$. The total derivative of K may be written as

$$dK = \frac{\partial K}{\partial Z}\, dZ + \frac{\partial K}{\partial(\Delta T_H)}\, d(\Delta T_H) + \frac{\partial K}{\partial(\Delta T_{WW})}\, d(\Delta T_{WW}) + \frac{\partial K}{\partial(\Delta T_{EC})} d(\Delta T_{EC}) \tag{29}$$

From equations (26) and (20), the equations of restraint are

$$\psi_1 = \Delta T_h - \Delta T_{WW} - \Delta T_{EC} = 0 \tag{30}$$

and

$$\psi = Z - 2 \sum_{j=1}^{5} \frac{(HTU)_j\, (\Delta i_y)_j}{I'_j + \frac{R_j}{2}(\Delta T_{EC}-5)} = 0 \tag{31}$$

Multiplying $d\psi_1$ by an unassigned multiplier λ_1 and $d\psi_2$ by a second unassigned multiplier λ_2 gives:

$$\lambda_1 \, [d(\Delta T_H) - d(\Delta T_{WW}) - d(\Delta T_{EC})] = 0 \tag{32}$$

and

$$\lambda_2 \, [dZ + (\sum_{j=1}^{5} \frac{(HTU)_j \, (\Delta i_j)_j \, R_j}{[\,(I_j') + \frac{R_j}{2}(\Delta T_{EC} - 5)]^2}) \, d(\Delta T_{EC})] = 0 \tag{33}$$

To describe the stationary points subject to the restraints (30) and (31), equation (29) is set equal to zero and added to equations (32) and (33) to give:

$$(\frac{\partial K}{\partial Z} + \lambda_2) \, dZ + (\frac{\partial K}{\partial \Delta T_H} + \lambda_1) \, d(\Delta T_H) + (\frac{\partial K}{\partial \Delta T_{WW}} - \lambda_1) \, d(\Delta T_{WW})$$

$$+ (\frac{\partial K}{\partial \Delta T_H} - \lambda_1 + \lambda_2 \sum_{j=1}^{5} \frac{(HTU)_j \, (\Delta i_y)_j \, R_j}{[I_j' + \frac{R_j}{2}(\Delta T_{EC} - 5)]^2}) \, d(\Delta T_{EC}) = 0 \tag{34}$$

To insure that equation (34) is identically zero, values are assigned to λ_1 and λ_2 such that the coefficient of each differential is zero. The four resulting equations are:

$$\frac{\partial K}{\partial Z} + \lambda_2 = 0 \tag{35}$$

$$\frac{\partial K}{\partial \Delta T_H} + \lambda_1 = 0 \tag{36}$$

$$\frac{\partial K}{\partial \Delta T_{WW}} - \lambda_1 = 0 \tag{37}$$

$$\frac{\partial K}{\partial \Delta T_{EC}} - \lambda_1 + \lambda_2 \sum_{j=1}^{5} \frac{(HTU)_j \, (\Delta i_y)_j \, R_j}{[I_j' + \frac{R_j}{2}(\Delta T_{EC} - 5)]^2} = 0 \tag{38}$$

These four equations along with equations (20) and (26) give the six equations necessary to solve for the six unknowns, λ_1, λ_2, Z, ΔT_H, ΔT_{WW}, and ΔT_{EC}.

Since ΔT_H appears in the equation for K to the first power, $\frac{\partial K}{\partial \Delta T_H}$ is a constant for a given water flow rate

$$\frac{\partial K}{\partial \Delta T_H} = 5.562 \times 10^{-6}\, W. \tag{39}$$

The value of λ_1 may now be obtained by substitution of equation (39) into equation (36).

$$\lambda_1 = - 5.562 \times 10^{-6} \, W \tag{40}$$

By equation (37)

$$\frac{\partial K}{\partial \Delta T_{WW}} = - 5.562 \times 10^{-6} \, W \tag{41}$$

and by equation (28)

$$\frac{\partial K}{\partial \Delta T_{WW}} = - 1.283 \times 10^{-2} \, (W)^{0.538} \, (\Delta T_{WW})^{-1.538}. \tag{42}$$

Combining equations (41) and (42) gives:

$$(\Delta T_{WW}) = 2310 \, (W)^{-0.462} \tag{43}$$

From equation (28)

$$\frac{\partial K}{\partial \Delta T_{EC}} = 2.002 \times 10^{-3} \sum_{j=1}^{5} \frac{C_j\ R_j\ (HTU)_j\ (\Delta i_y)_j}{[I'_j + \frac{R_j}{2}(\Delta T_{EC} - 5)]^2} \tag{44}$$

and

$$\frac{\partial K}{\partial Z} = 1.787 \times 10^{-8}\ W + \frac{0.01365\ M\ f_p}{(29.4 + Z\ f_p)^{0.71}\ (29.4 - Z\ f_p)^{1.29}}$$

$$+ \frac{3.868^{-5}\ (W)^{0.591}}{Z^{0.409}} + \frac{0.291\ f_p\ (M)^{0.725}\ (29.4 - Zf_p)^{-1.29}}{[(\frac{29.4 + Zf_p}{29.4 - Zf_p})^{0.29} - 1]^{0.725}\ (29.4 + Zf_p)^{0.71}} \tag{45}$$

Equation (38) may now be written in terms of two variables Z and ΔT_{EC} as

$$0 = -2.002 \times 10^{-3} \sum_{j=1}^{5} \frac{C_j\ R_j\ (HTU)_j\ (\Delta i_y)_j}{[I'_j + \frac{R_j}{2}(\Delta T_{EC} - 5)]^2} + 5.562 \times 10^{-6}\ W$$

$$-[1.787 \times 10^{-8}\ W + \frac{0.01365\ M\ f_p\ (29.4 - Zf_p)^{-1.29}}{(29.4 + Zf_p)^{0.71}} + \frac{3.868 \times 10^{-5} (W)^{0.591}}{(Z)^{0.409}}$$

$$+ \frac{0.291\ f_p\ (M)^{0.725} (29.4 - Zf_p)^{-1.29}}{[(\frac{29.4+Zf_p}{29.4-Zf_p})^{0.29} - 1]^{0.725} (29.4+Zf_p)^{0.71}}][\sum_{j=1}^{5} \frac{(HTU)_j (\Delta i_y)_j\ R_j}{[I'_j + \frac{R_j}{2}(\Delta T_{EC} - 5)]^2}] \tag{46}$$

Equation (46) and (31) are two independent equations in the two unknowns, ΔT_{EC} and Z. Their solution was found by trial and substitution with the aid of a Burroughs 220 computer. Four cases were considered corresponding to four capacities: 10,000 g.p.d.; 100,000 g.p.d.; 1,000,000 g.p.d.; and 10,000,000 g.p.d.

Optimum values of ΔT_{EC} and ΔT_{WW} from equations (43), (46) and (31) were then substituted into equation (26) to obtain the optimum values of ΔT_H for the four cases.

CHAPTER IV

DISCUSSION OF RESULTS

Range of Results.-- Optimum values of the variables investigated were determined for plant capacities ranging from 10,000 gallons per day to 10,000,000 gallons per day. The solution for a smaller plant capacity (1,000 g.p.d.) was seen to be out of the range of practical designs and was not pursued to convergence.

Response of Optimum Values and Cost to Plant Size.-- The effect of plant capacity on the optimum operating conditions is shown in Figure 6. One would expect the relative importance of capital charges to decrease with increasing plant capacity since unit equipment costs usually decrease with an increase in equipment size. On the other hand, operating costs are more nearly proportional to plant capacity. Since the temperature potentials ΔT_{WW} and ΔT_{EC} are indicative of equipment size one would expect their optimum values to decrease with increasing plant capacity. Figure 6 shows that this is indeed the case. The ΔT_{EC} decreases from $19.08^{\circ}F$ to $7.141^{\circ}F$ over the range of plant capacities from 10^4 gallons per day to 10^7 gallons per day. Over the same range ΔT_{WW} decreased from $5.53^{\circ}F$ to $0.70^{\circ}F$.

It may be noted that the cost of the water-to-water heat exchanger is so insignificant for the largest plant size studied that its optimum temperature potential becomes extremely small ($0.70^{\circ}F$) and probably falls below the range of practical operation.

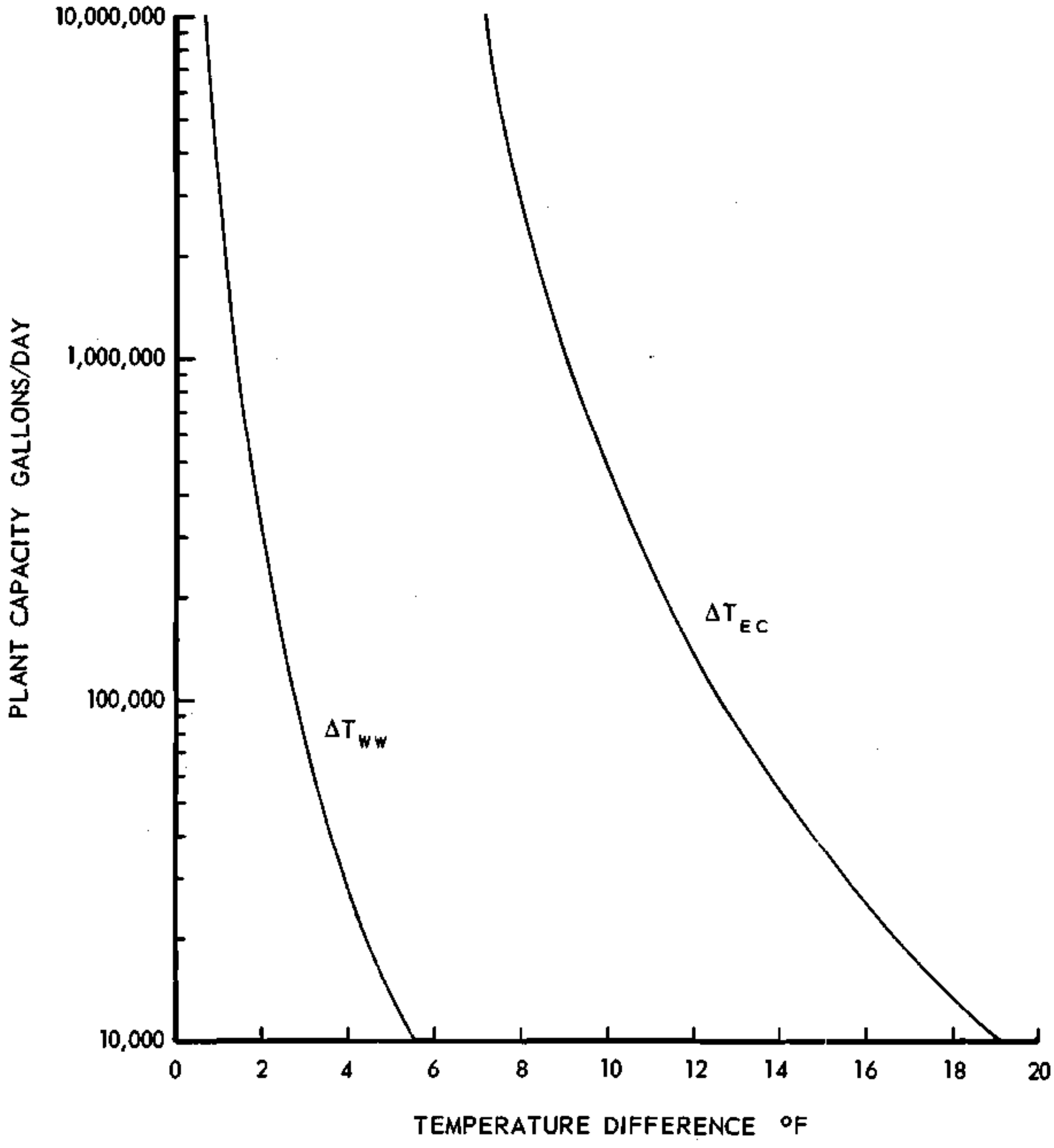

Figure 6. Effect of Plant Size on Optimum Conditions.

Figure 7 shows the effect of plant capacity on the final unit cost of water conversion. The curve shows that as the plant size decreases, the unit cost of conversion increases at a rapidly increasing rate.

It is interesting to note that the unit cost of water conversion for the optimum design of the 100,000 gallon per day plant is $0.08 less than a previously determined cost (11) for a plant of the same size and basic design believed to be near the optimum.

Table 5 lists the optimum values of the variables, the unit cost of water conversion, and the optimum number of effects for each of the four cases studied.

Table 5 Optimum Values and Unit Costs

Plant Size (g.p.d.)	ΔT_H (^oF)	ΔT_{EC} (^oF)	ΔT_{WW} (^oF)	Z (ft)	Number of Effects	Unit Costs ($/1,000 gal.)
10,000	24.6	19.1	5.53	50.5	2.56	1.63
100,000	15.4	12.6	2.77	76.6	4.09	1.07
1,000,000	10.4	9.01	1.37	106.0	6.06	0.76
10,000,000	7.84	7.14	0.70	135.3	8.04	0.61

<u>Limitations of Results</u>.-- The reader is cautioned that the results obtained by the foregoing techniques are valid only for the assumptions made. Many of the factors considered as constants would be affected by not only plant size but also by geographical location, proximity to other industry, availability of waste heat, concentration of original polluted water, and by far the most important, variations of the basic design.

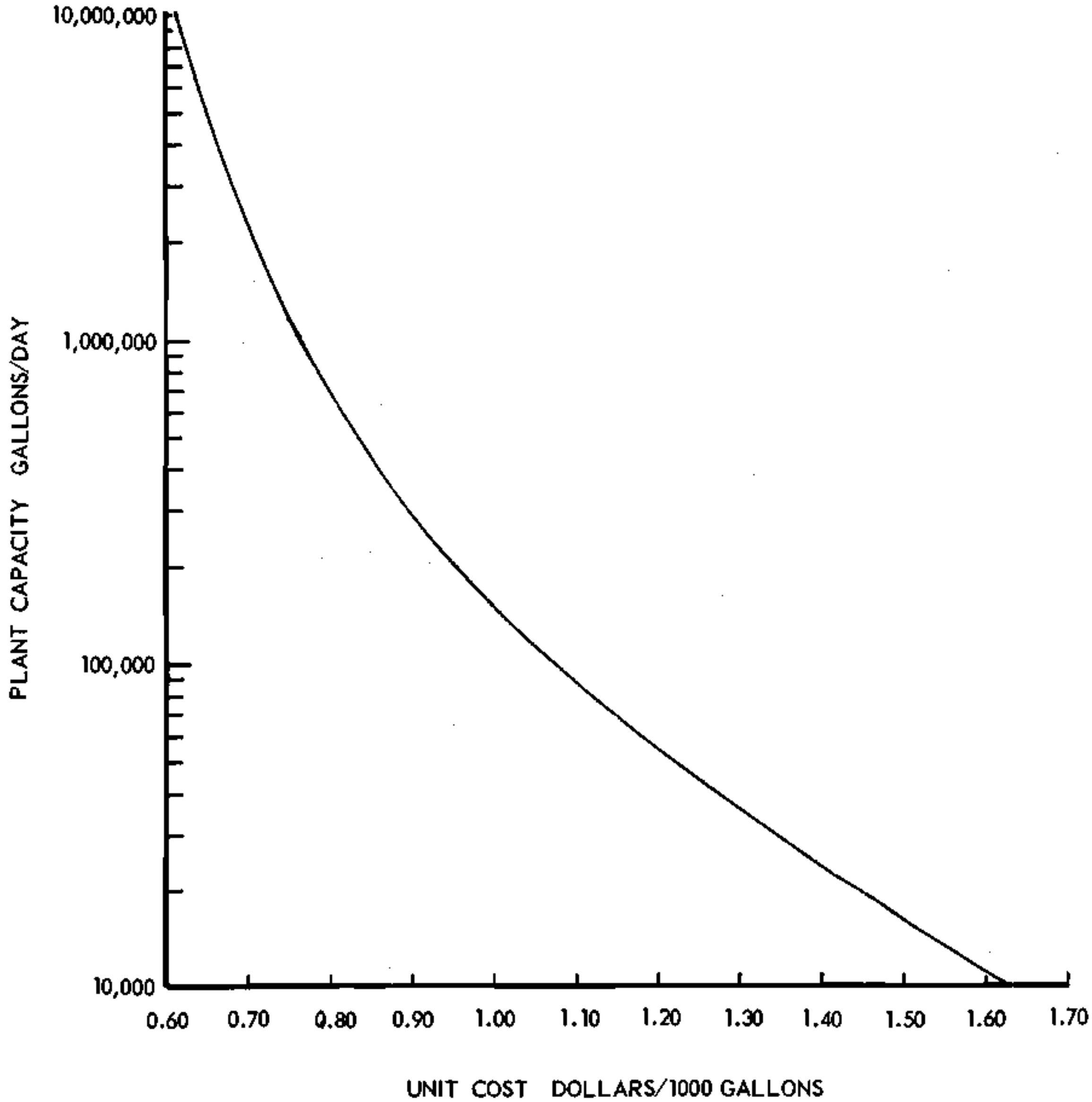

Figure 7. Effect of Plant Size on Unit Cost of Water Conversion.

For the purpose of orienting the results a list of the major assumptions follows. It was assumed that:

1. the cost estimating procedure set forth by the Office of Saline Water, Department of Interior, is valid when applied to this process;

2. the cost of steam is not a function of plant size;

3. the heat transfer coefficient for the water-to-water heat exchanger is constant;

4. the pressure drop in the water-to-water heat exchanger is directly proportional to the first power of the total column height (Z);

5. the saturation equilibrium line is not materially affected by deviations of pressure encountered in the operation of this cycle;

6. the adiabatic turbine efficiency is 80 per cent;

7. the blower operation is adiabatic;

8. one large blower placed between section 5 of the condenser and section 5 of the evaporator is used in lieu of the ten small blowers placed as shown in Figure 3; and

9. the heat and mass transfer coefficients for MASPAC FN-200 are of the same form as the coefficients for Raschig rings.

It is realized that some of these assumptions are subject to question. However, they were made in the interest of simplifying the presentation of this technique to a manageable form.

CHAPTER V

CONCLUSIONS

From the results obtained in this investigation it may be concluded that:

1. It is possible to formulate sufficient general expressions to mathematically determine an optimum set of operating conditions.

2. Owing to practical considerations, the cycle as presently envisioned would not be operated at the optimum value for ΔT_{WW} in the large plant sizes.

3. The unit cost of water conversion may be reduced below the value ($1.00/1,000 gal.) commensurate with the present state of the art.

CHAPTER VI

RECOMMENDATIONS

1. It is recommended that an attempt be made to experimentally verify the validity of the assumptions made in this study, particularly the assumption concerning the form of the expression for the heat and mass transfer coefficients (assumption No. 9).

2. It is further recommended that the effect of cycle operation at other than atmospheric pressure be investigated either experimentally or analytically.

3. The optimum operating conditions for the design shown in Figure 1 should be determined for comparison with the results of this study.

A P P E N D I X

GRAPHICAL DETERMINATION OF THE INTEGRAL REPRESENTING THE NUMBER OF TRANSFER UNITS

The value of I in equation (8) was determined for the base design by plotting a series of lines, with a slope given by equation (5), on the enthalpy-temperature plane. A series of values for $(i_i - i_y)$ was measured as the vertical distance between the intersections of equation (5) with the operating line and the saturation line as shown in Figure 8. Values measured from Figures 8, 9, 10, 11 and 12 are listed in Table 3.

Since the values of $(i_i - i_y)$ are nearly symmetrical about their median value and since the per cent variation is not large in any one section the value of I may be taken as the reciprocal of the average of the reciprocals of $(i_i - i_y)$. This is approximately equivalent to plotting the reciprocals of $(i_i - i_y)$ for the various values of i_y, graphically integrating, and dividing by $(i_{y_2} - i_{y_1})$. Average values of I are listed in Table 4 in the column labeled I_j'.

Table 3 Values of Enthalpy Potential for Base Design

i	Condenser $1/(i_i - i_y)$	Evaporator $1/(i_i - i_y)$
	Section 1	
2677	0.01538	0.01020
2600	0.01370	0.01087
2500	0.01282	0.01205
2400	0.01220	0.01282
2300	0.01163	0.01389
2200	0.01149	0.01471
2100	0.01163	0.01563
2000	0.01163	0.01538
1900	0.01190	0.01515
1800	0.01282	0.01471
1700	0.01389	0.01389
1600	0.01613	0.01282
1500	0.02000	0.01163
Avg.	0.01348	0.01337
	Section 2	
1500	0.03030	0.01852
1460	0.02632	0.02000
1420	0.02381	0.02128
1380	0.02222	0.02381
1340	0.02128	0.02564
1300	0.02041	0.02778
1260	0.01961	0.03030
1220	0.01923	0.03333
1180	0.01923	0.03448
1140	0.01923	0.03571
1100	0.01961	0.03571
1060	0.02000	0.03571
1020	0.02083	0.03448
980	0.02222	0.03226
940	0.02381	0.03125
900	0.02778	0.02778
860	0.03125	0.02632
820	0.03704	0.02381
800	0.04347	0.02222
Avg.	0.02461	0.02844
	Section 3	
800	0.06061	0.03333
790	0.05556	0.03571
770	0.04878	0.03846
750	0.04444	0.04348
730	0.04167	0.04762
710	0.03922	0.05263
690	0.03846	0.05714
670	0.03704	0.06250
650	0.03636	0.06667
630	0.03636	0.06897
610	0.03704	0.07407
590	0.03704	0.07143
570	0.03846	0.07143
550	0.04000	0.07143
530	0.04348	0.06667
510	0.04651	0.06250
490	0.05263	0.05555
470	0.06250	0.05000
450	0.07692	0.04348
Avg.	0.04595	0.05648
	Section 4	
450	0.10000	0.05714
440	0.08696	0.06061
420	0.07407	0.07407
400	0.06667	0.09091
380	0.06250	0.10526
360	0.06061	0.11765
340	0.06250	0.13333
320	0.06666	0.12500
300	0.07407	0.11111
280	0.08696	0.09524
270	0.10000	0.09091
Avg.	0.07645	0.09648
	Section 5	
270	0.11760	0.10200
260	0.11110	0.11760
250	0.10530	0.12500
240	0.10530	0.13790
230	0.10530	0.14290
220	0.11110	0.14290
210	0.11760	0.13790
200	0.13330	0.13160
190	0.15380	0.11900
Avg.	0.11780	0.12850

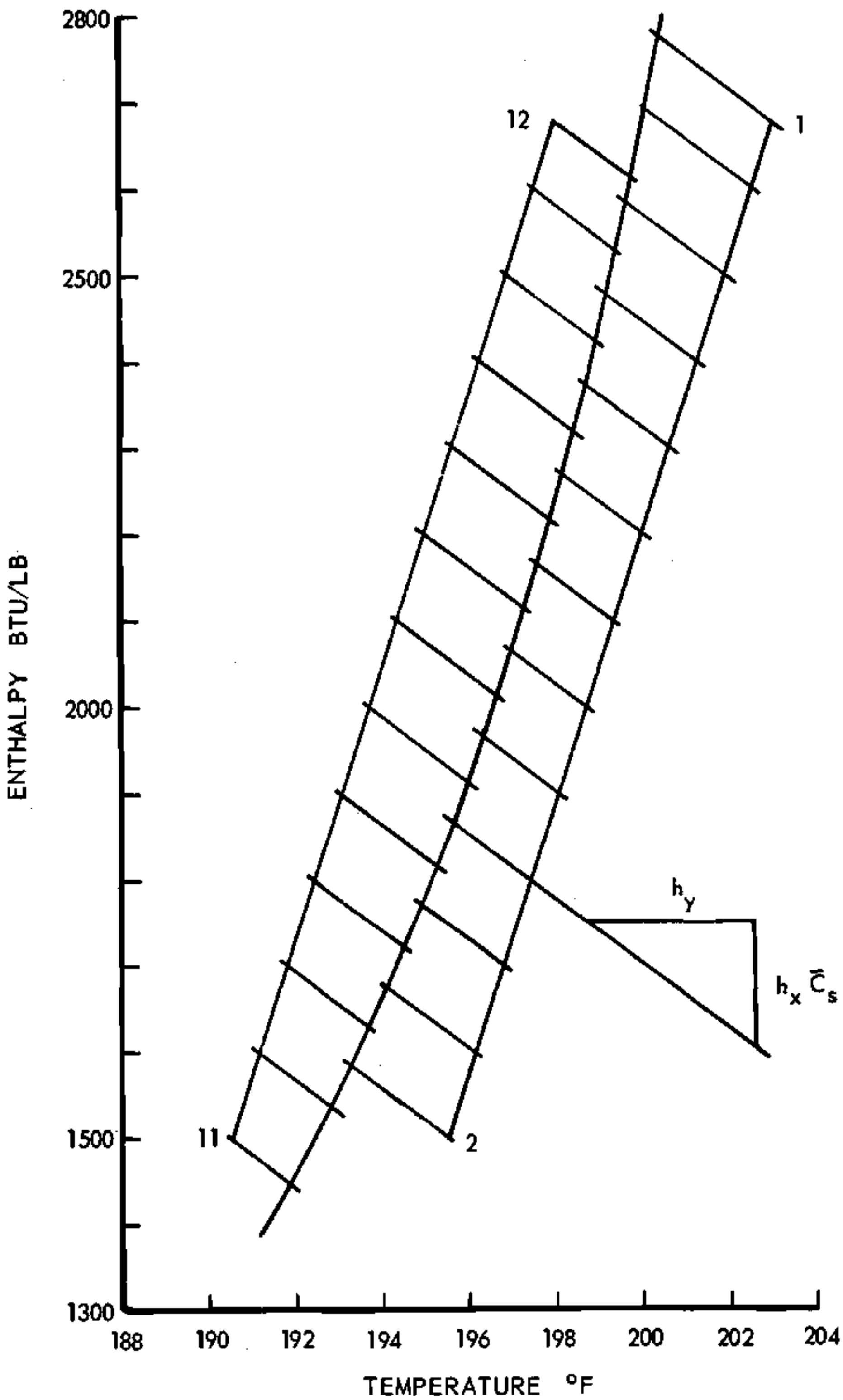

Figure 8. Graphical Determination of Enthalpy Potential for Section 1.

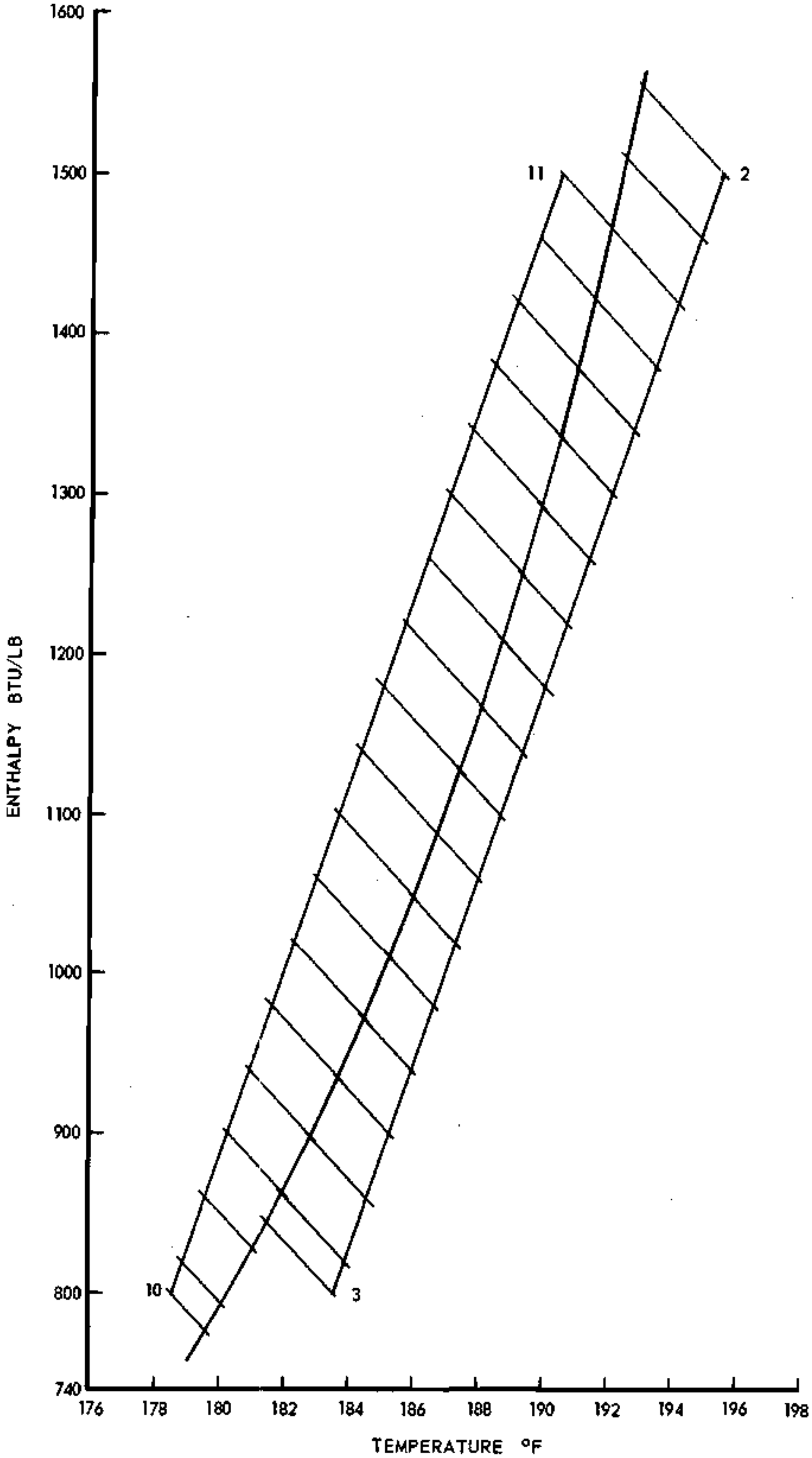

Figure 9. Graphical Determination of Enthalpy Potential for Section 2.

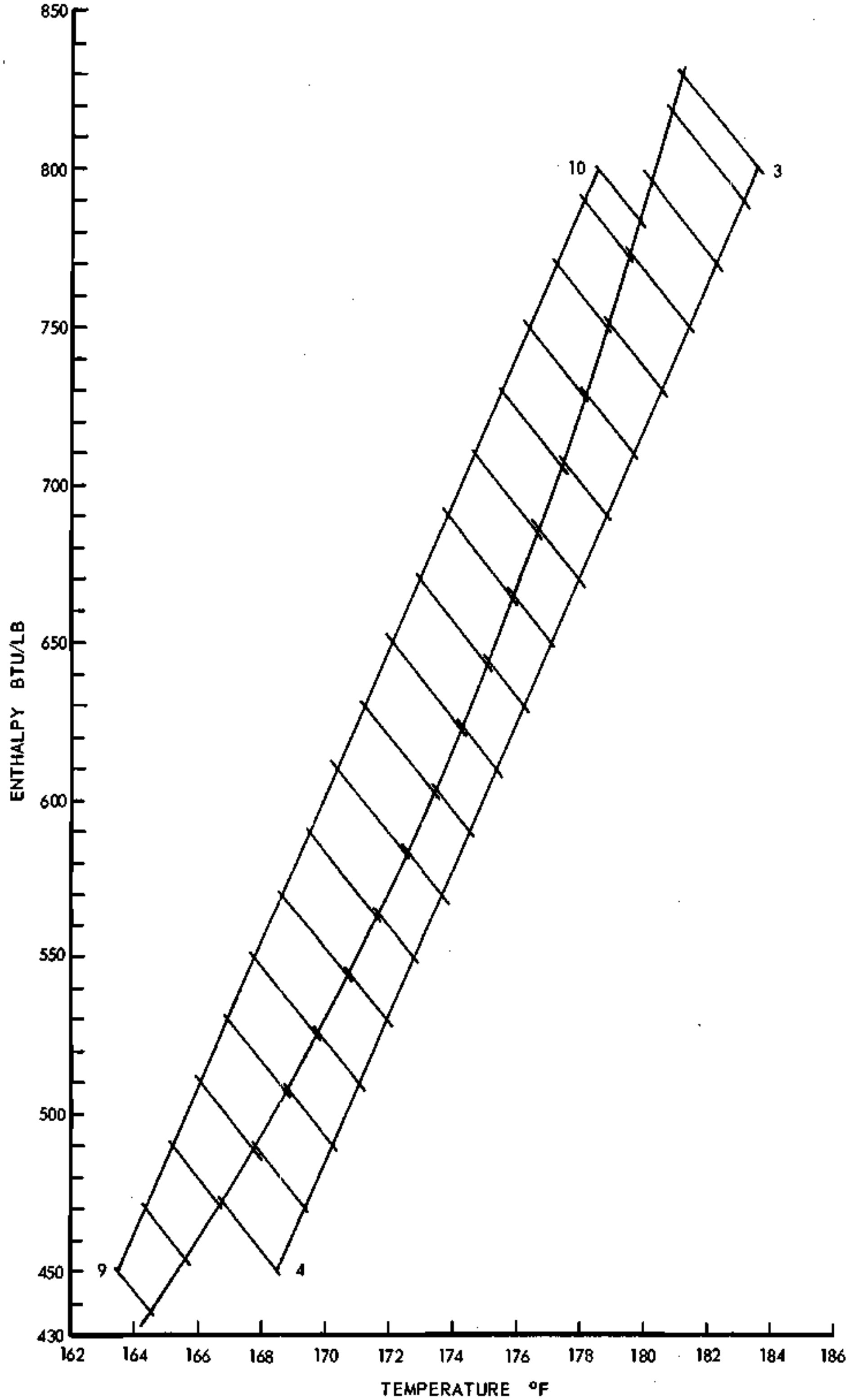

Figure 10. Graphical Determination of Enthalpy Potential for Section 3.

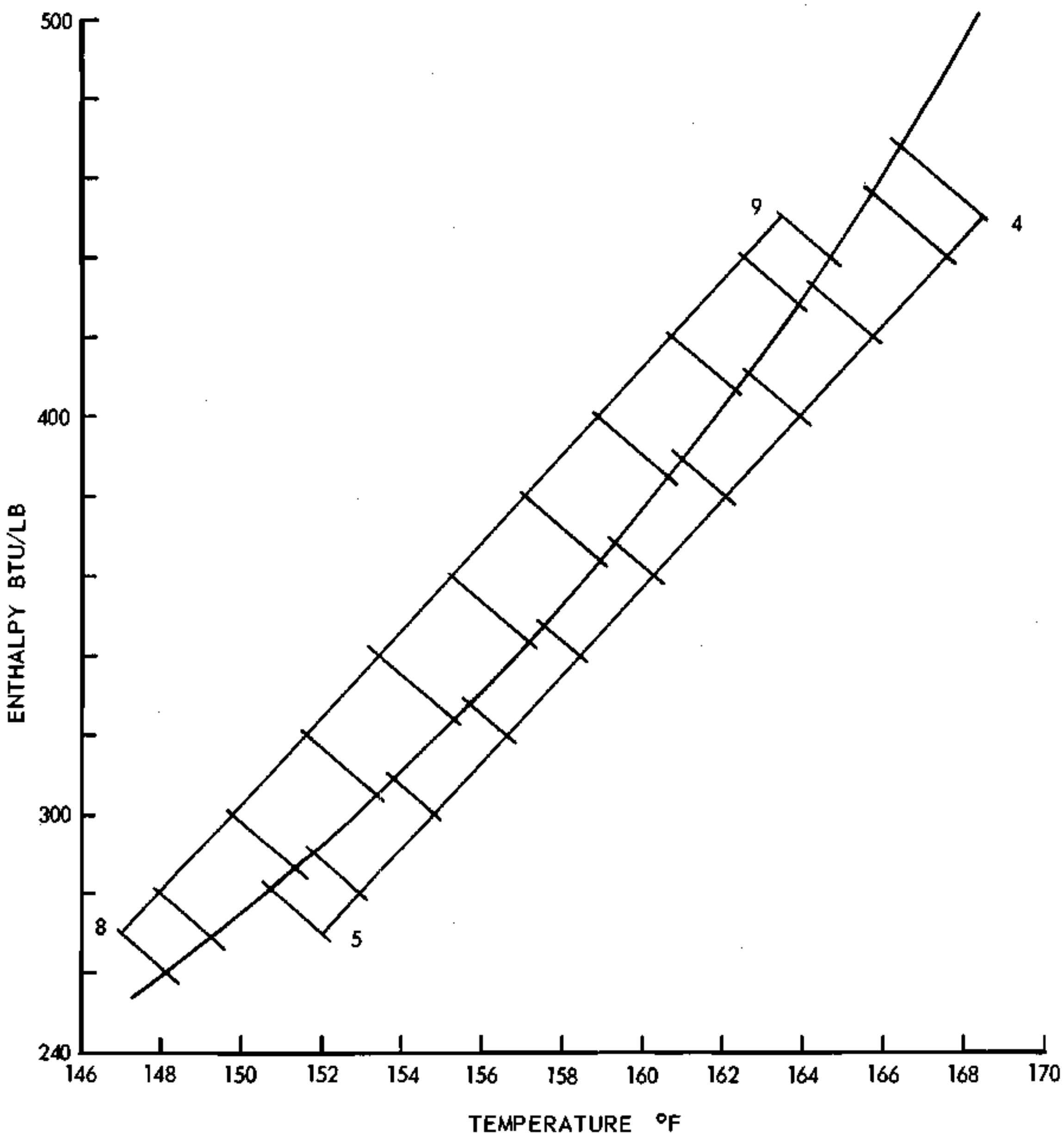

Figure 11. Graphical Determination of Enthalpy Potential for Section 4.

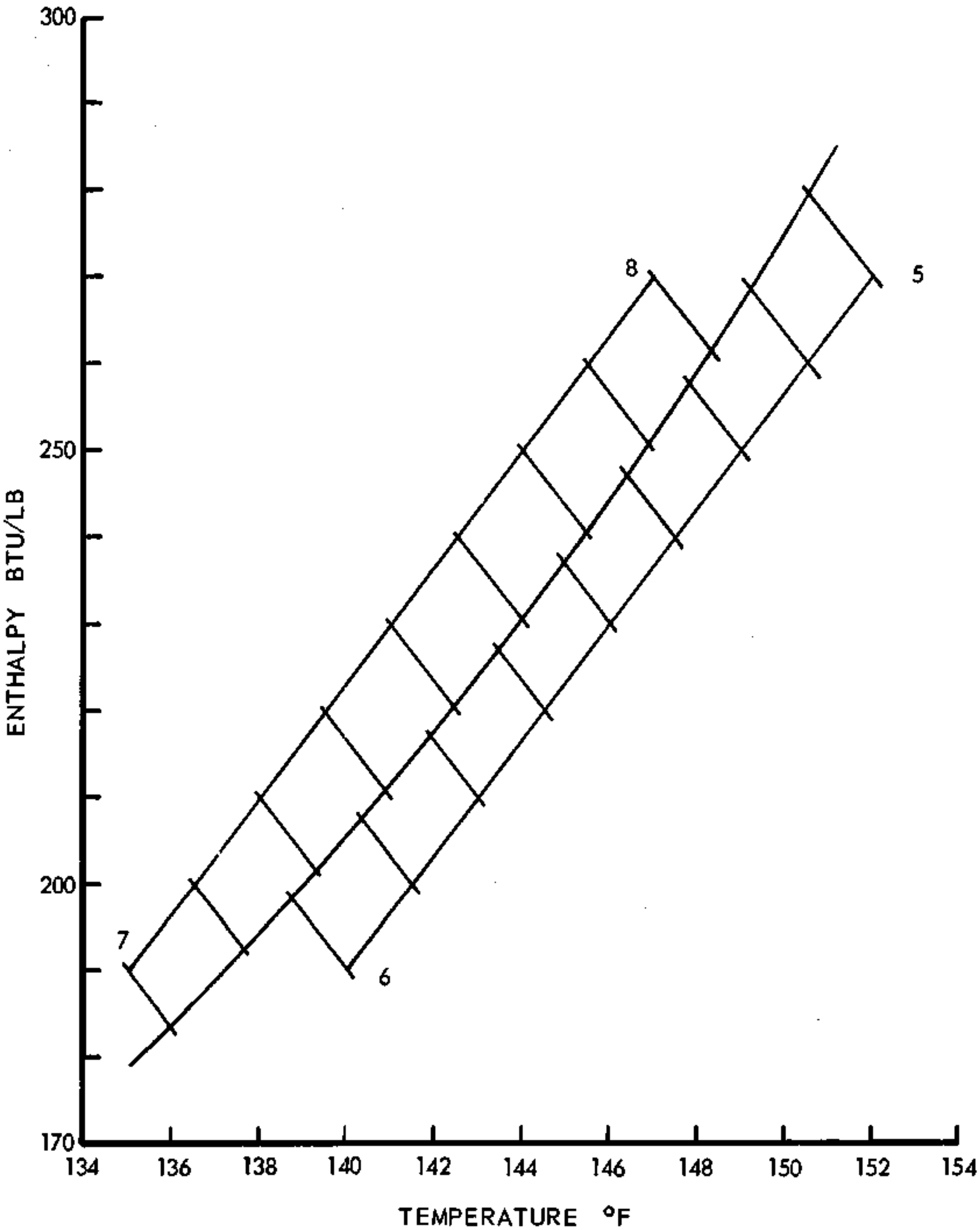

Figure 12. Graphical Determination of Enthalpy Potential for Section 5.

COST EQUATIONS

The total daily cost of water conversion (K) may be expressed in equation form equivalent to the standard procedure for estimating costs set forth by the Office of Saline Water (10). In the case of the humidity cycle, K may be expressed as the sum of ten individual cost contributions (X_i) including the following:

X_1 = daily cost of electric power,

X_2 = daily steam consumption x unit steam cost,

X_3 = daily cost of supplies and maintenance materials

= 1.5×10^{-5} (TPI),

X_4 = operating labor costs = $0.1\ (X_1 + X_2 + X_3 + X_8)$,

X_5 = maintenance labor costs = 1.5×10^{-5} (TPI),

X_6 = payroll extras = $0.15\ (X_4 + X_5)$,

X_7 = general and administrative overhead = $0.30\ (X_4 + X_5 + X_6)$,

X_8 = amortization costs* = 2.24×10^{-4} (TPI),

X_9 = taxes and insurance = 6.0×10^{-5} (TPI), and

X_{10} = interest on working capital = 7.25×10^{-5}

$(X_1 + X_2 + \dots + X_9)$,

where (TPI) is the total plant investment.

Addition and simplification of the above cost contributions give an expression for K:

---

*Amortization is based on 4% interest compounded annually and a 20-year life for all equipment.

$$K = 1.1578 \, (X_1 + X_2) + 3.5974 \times 10^{-5} \, (TPI) \tag{A-1}$$

The total plant investment may be expressed in terms of the following ten items:

Y_1 = special equipment,

Y_2 = standard engineering equipment,

Y_3 = erection and assembly of plant = $0.3 \, (Y_1 + Y_2)$,

Y_4 = instruments = $0.04 \, (Y_1 + Y_2)$,

Y_5 = raw water supply = 5.0×10^{-3} P,

Y_6 = product water storage = 1.0×10^{-2} P,

Y_7 = contingencies = $0.1 \, (Y_1 + \cdots + Y_6)$,

Y_8 = engineering = $0.1 \, (Y_1 + \cdots + Y_7)$,

Y_9 = interest on investment during construction = $0.04 \, (Y_1 + \cdots + Y_8)$,

and Y_{10} = site = 6.0×10^{-3} P.

Addition and simplification of the above items give an expression for TPI:

$$TPI = 1.6863 \, (Y_1 + Y_2) + 2.488 \times 10^{-2} \, P \tag{A-2}$$

Substitution of equation (A-2) into equation (A-1) gives the final general form of the cost equation:

$$K = 1.158 \, (X_1 + X_2) + 6.065 \times 10^{-4} \, (Y_1 + Y_2) + 0.895 \times 10^{-5} \, P \tag{A-3}$$

The total steam consumed per day is the sum of the steam used in the turbine for pumps and blower and the steam used to heat water:

$$M_{St_T} = M_{St_B} + M_{St_p} + M_{St_H} \tag{A-4}$$

The steam required in the turbine for blower operation M_{St_B} during a 24 hour period is a function of the horsepower of equation (10), the energy available in a pound of steam, and a turbine efficiency.

$$M_{St_B} = \frac{(2545)(24)\ (hp)_B}{\eta \quad i_{SS}} \tag{A-5}$$

where η is the turbine efficiency and i_{SS} is energy available in one pound of steam for expansion from a superheated condition to a saturated condition at one atmosphere.

Likewise, the steam required in the turbine for pump operation is:

$$M_{St_p} = \frac{(2545)(24)(hp)_p}{\eta \quad i_{SS}} \tag{A-6}$$

If the steam exhausted from the turbine is used in the water heater along with additional superheated steam, the quantity of additional superheated steam required for water heating is:

$$M_{St_H} = [24\ W\ (\Delta T_H) - (M_{St_p} + M_{St_B})(i_S - i_{SS})]/i_S \tag{A-7}$$

where the specific heat of water has been taken as unity.

Substituting the expressions for horsepower from equations (10) and (11) into the sum of equations (A-5)(A-6) and (A-7) gives the expression for total steam requirement over a 24 hour period:

$$M_{St_T} = \frac{1}{\eta\, i_S} \left(0.0617\ W\ Z + 15.38\ M\ p_1\ v_1\ \left[\left(\frac{14.7 + \frac{Zf_p}{2}}{14.7 - \frac{Zf_p}{2}}\right)^{0.29} - 1\right] + 24\ W\ \Delta T_H\right) \tag{A-8}$$

where the total pressure head for the pumps is assumed to be two times the total column height (Z).

The cost of the packed columns is the product of the total volume of the columns and the cost per unit volume. The volume may be expressed in terms of the length determined by equation (20) and the cross section area.

$$V_C\, C_C = 6.6\, W \sum_{j=1}^{5} \frac{(HTU)_j\, (\Delta i_y)_j\, C_j}{I'_j + \frac{R_j}{2}(\Delta T_{EC}-5)} \tag{A-9}$$

where the area is taken as

$$A_j = \frac{W}{(G_x)_j} = W\, C_j \tag{A-10}$$

Expressions for $A_{WW}\, C_{WW}$, $(hp)_p\, (C_{hp})_p$, and $(hp)_B\, (C_{hp})_B$ were formulated from cost data presented in the appendix of the standard cost estimating procedure by the Office of Saline Water (10). The data were presented in the form of curves from which the following expressions were taken:

$$A_{WW}\, C_{WW} = 81.2\, (A_{WW})^{0.538} \tag{A-11}$$

$$(hp)_p\, (C_{hp})_p = 377\, (hp)_p^{\,0.591} \tag{A-12}$$

$$(hp)_B\, (C_{hp})_B = 505\, (hp)_B^{\,0.725} \tag{A-13}$$

Equation (A-13) is a linear approximation of a slightly curved plot on log-log coordinates while equations (A-11) and (A-12) are exact fits of straight lines on a log-log plot.

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